Reg. No. | | | | | | | | |

BSCMTC 253

Credit Based IV Semester B.Sc. Examination, April /May 2017
(2015 – 16 Batch Onwards) (Semester Scheme)
MATHEMATICS – IV
Multiple Integrals, Infinite Sequences and Series and Complex Analysis

Time : 3 Hours Max. Marks : 120

***Note** : A single answer booklet containing **40** pages will be issued. **No** additional sheets will be issued.*

***Instructions** : 1) Answer **any ten** questions from Part **A**. **Each** question carries **3** marks.*
*2) Answers to Part **A** should be written in the first **four** pages of the **main** answer book.*
*3) Answer **five full** questions from Part **B** choosing one **full** question from each Unit.*
*4) Scientific calculators are **allowed**.*

PART – A

Answer **any ten** questions. **(3×10=30)**

1. Sketch the region of integration for the integral $\int_0^1 \int_2^{4-2x} dydx$ and write an equivalent integral with order of integration reversed.
2. Find the average value f (x, y) = x cos xy over the rectangle R : $0 \le x \le \pi$, $0 \le y \le 1$.
3. Evaluate $\int_0^1\int_0^1\int_0^1 (x^2 + y^2 + z^2)$ dzdydx.
4. Write $(1 - i\sqrt{3})^{72}$ in the form x + iy.
5. Find the domain and range of $f(z) = \frac{iz}{z - \bar{z}}$.
6. Find the singular points of $f(z) = \frac{z^3 - 1}{z(z^2 - 1)}$.

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7. Prove that the real and imaginary parts of an analytic function are harmonic.
8. Show that $\sin iy = i \sinh y$.
9. Find the general value of $\log\left(-1+i\sqrt{3}\right)$.
10. Apply the definition to show that $\lim_{n\to\infty}\frac{1}{n}=0$.
11. Find $\lim_{n\to\infty}\frac{5^n}{7^n}$.
12. Find the Taylor polynomial of order two generated by $f(x)=\frac{1}{x}$ at $a = 2$.
13. Find whether the series $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ converges.
14. If $\sum_{n=1}^{\infty}|a_n|$ converges, then prove that $\sum_{n=1}^{\infty} a_n$ converges.
15. Find whether the series $\sum_{n=1}^{\infty}\frac{n^2}{2^n}$ converges.

PART – B

Unit – I

1. a) Integrate $f(u, v) = v - \sqrt{u}$ over the triangular region cut from the first quadrant of the uv – plane by the curve $u + v = 1$.
   b) Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and below by the triangle enclosed by the lines $y = x$, $x = 0$, and $x + y = 2$ in the xy – plane.
   c) Find the area enclosed by the cardioid $r = a(1 + \cos\theta)$. **(6+6+6)**
2. a) Integrate $f(x, y) = x^2 + y^2$ over the triangular region with vertices (0, 0), (1, 0) and (0, 1).
   b) Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$ using double integrals.
   c) Integrate $f(x, y) = \frac{\ln(x^2+y^2)}{\sqrt{x^2+y^2}}$ over the region $1 \le x^2 + y^2 \le e$, by converting it into a polar integral. **(6+6+6)**

Unit – II

3. a) Using the definition of limit, show that $\lim_{z \to z_0} (az^2 + bz + c) = az_0^2 + dz_0 + c$.

b) Define $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$

Show that C.R. equations are satisfied at $z = 0$, but $f(z)$ is not differentiable at $z = 0$.

c) Show that $f(x, y) = e^x (\cos x - i \sin y)$ is nowhere analytic. **(6+6+6)**

4. a) Find the square roots of $-5 - 12i$.

b) Let $f(z) = u(x,y) + iv(x, y)$ be defined in some neighbourhood of $z = a + ib$. If the first order partial derivatives of u and v are continuous at (a, b), and if they satisfy the Cauchy Riemann equations, prove that $f'(z)$ exists at $z = a + ib$.

c) Show that $f(z) = \frac{1}{z-1}$ is analytic at $z = 1 + i$. **(6+6+6)**

Unit – III

5. a) Find the period and all the roots of the function $\sinh(iz + 2)$.

b) Solve the equation $e^{2z-1} = 1 + i$.

c) Does $\lim_{z \to \infty} e^{-z^2}$ exist ? Explain. **(6+6+6)**

6. a) Prove that $\sinh(z_1 + z_2) = \sinh z_1 \cosh z_2 + i \cosh z_1 \sinh z_2$.

b) Show that $\text{Log}\left(1 + i\sqrt{3}\right)^2 = 2\,\text{Log}\left(1 + i\sqrt{3}\right)$ but

$\text{Log}\left(-1 + i\sqrt{3}\right)^2 \neq 2\,\text{Log}\left(-1 + i\sqrt{3}\right)$.

c) Find $\int_0^{1+2i} \left(3x^2 - y + ix^3\right) dz$ along the real axis from $z = 0$ to $z = 1$, and along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + 2i$. **(6+6+6)**

Unit – IV

7. a) Discuss the convergence of $a_n = \left(\frac{3n+1}{3n-1}\right)^n$.

   b) Find whether the following series converge. If so find their sum.

   i) $\sum_{n=0}^{\infty}\left(\frac{5}{2^n}+\frac{1}{3^n}\right)$ ii) $\sum_{n=0}^{\infty} ln\left(\frac{n}{n+1}\right)$.

   c) Find the Taylor series generated by $f(x) = x^3 - 2x + 4$, at $a = 2$. **(6+6+6)**

8. a) Test the convergence of the sequence whose n^{th} term is

   i) $\sqrt{\frac{2n}{n+1}}$ ii) $\frac{(-1)^{n+1}}{2n-1}$

   b) Express the repeating decimal 0.234234 ... as a ratio of the two integers.

   c) Find the Maclaurin series for $f(x) = \sin 3x$ by deriving Taylor series of $\sin x$. **(6+6+6)**

Unit – V

9. a) State and prove the integral test for convergence of series.

   b) Test the convergence of

   i) $\sum_{n=1}^{\infty} \frac{ln\, n}{n^{\frac{3}{2}}}$ ii) $\sum_{n=1}^{\infty} n! e^{-n}$.

   c) Apply the absolute convergence to

   i) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+3}$ ii) $\sum_{n=1}^{\infty} \frac{(-100)^n}{n!}$ **(6+6+6)**

10. a) State and prove ratio test for convergence of series.

    b) Test the convergence of

    i) $\sum_{n=1}^{\infty} \frac{ln\, n}{n}$ ii) $\sum_{n=1}^{\infty}\left(1-\frac{3}{n}\right)^n$.

    c) State and prove the Leibnitz's Theorem for convergence of alternating series. **(6+6+6)**

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