Reg. No. | | | | | | | | |

BSCMTC 352

Credit Based VI Semester B.Sc. Examination, April/May 2017
(2013-14 and Earlier Batches)
(Semester Scheme)
MATHEMATICS (Paper – VIII a) (Special Paper)
Graph Theory

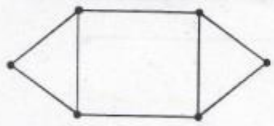
Time : 3 Hours

Max. Marks : 120

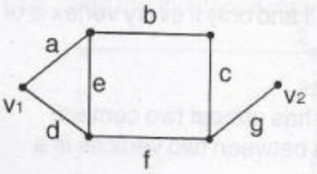
Instructions : *1) Answer **any ten** questions from Part **A**. **Each** question carries **3** marks.*
*2) Answer **five full** questions from Part **B** choosing **one full** question from **each** Unit.*
*3) Scientific calculators are **allowed**.*

PART – A

1. Prove that the maximum number of edges in a simple graph is $\frac{n(n-1)}{2}$. **(3×10=30)**
2. In a binary tree of n vertices, prove that the number of pendant vertices is $p = \frac{n+1}{2}$.
3. Prove that there are atleast two pendant vertices in a tree.
4. Write the vertex connectivity and edge connectivity of the following graph.

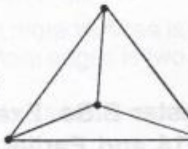


5. Write two similarities of K_5 and $K_{3,3}$ graphs.
6. If G is a simple connected planar graph with n vertices, e edges and f regions, prove that $2e \geq 3f$.
7. Write incidence matrix of a complete graph with 5 vertices.
8. Write the path matrix $P(v_1, v_2)$ for the vertex pair v_1, v_2 in the graph.

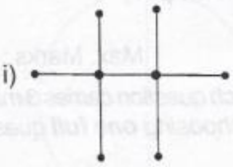
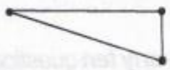


9. Prove that reduced incidence matrix of a tree is non-singular.

P.T.O.

10. Write the chromatic polynomial of 

11. Find the chromatic number of the graphs

i)  ii) 

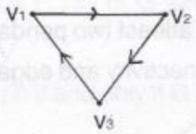
12. Define the terms

i) properly coloring of a graph

ii) chromatic number of a graph.

13. Define balanced digraph and give an example.

14. Define the term arborescence and draw an arborescence with three vertices.

15. Write the adjacency matrix of the digraph 

PART – B

UNIT – I

1. a) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. **6**

b) Prove that a graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges. **6**

c) Prove that a connected graph is an Euler graph if and only if every vertex is of even degree. **6**

2. a) Prove that a tree with n vertices has n – 1 edges. **6**

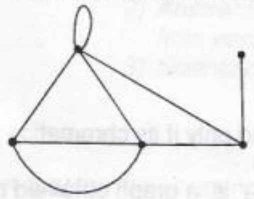
b) Define center of a graph. Prove that every tree has atmost two centers. **6**

c) Define distance in a graph. Prove that distance between two vertices in a graph is a metric. **6**

UNIT – II

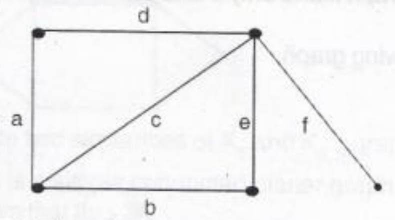
3. a) Prove that in a graph, every circuit has an even number of edges in common with any cutset. **6**
   b) Prove that K_5 is nonplanar. **6**
   c) Prove that in a connected graph any minimal set of edges containing atleast one branch of every spanning tree is a cutset. **6**

4. a) Prove that a connected planar graph with n vertices, e edges has $e - n + 2$ regions. **6**
   b) Prove that a graph is planar if and only if it can be embedded on the surface of a sphere. **6**
   c) Draw the Geometric dual of the graph. **6**

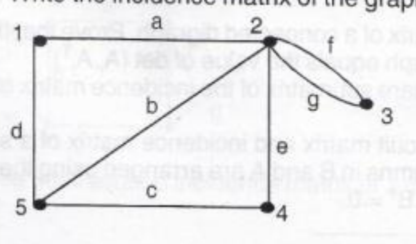


UNIT – III

5. a) Prove that the rank of the incidence matrix of a connected graph with n vertices is $n - 1$. **6**
   b) Prove that in a graph, ring sum of two circuits is either a circuit or an edge disjoint union of circuits. **6**
   c) Write the cutset matrix for the following graph. **6**



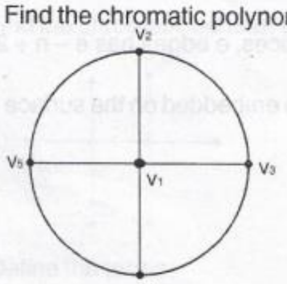
6. a) If B is a circuit matrix of a connected graph G with e edges and n vertices, prove that rank of B is $e - n + 1$. **6**
   b) Prove that in a graph with n vertices, rank of a cutset matrix is $n - 1$. **6**
   c) Write the incidence matrix of the graph. **6**



UNIT – IV

7. a) Prove that every tree with two or more vertices is two chromatic. 6
b) Prove that a graph with one or more edges is two chromatic if and only if it has no circuit of odd length. 6
c) Prove that a graph with n vertices is a tree if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$. 6

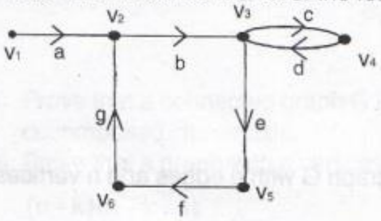
8. a) Find the chromatic polynomial for the graph. 6



b) Prove that a graph with n vertices is complete if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda-1)(\lambda-2)\ldots(\lambda-n+1)$. 6
c) If a and b are non adjacent vertices in a graph G, G′ is a graph obtained by adding an edge between a and b in G and G″ is a simple graph obtained by fusing the vertices a and b in G and replacing the set of parallel edges by a single edge in G. Prove that $P_n(\lambda)$ of G = $P_n(\lambda)$ of $G' + P_{n-1}(\lambda)$ of G″. 6

UNIT – V

9. a) Prove that a digraph G is an Euler digraph if and only if G is connected and balanced. 6
b) Write the incidence matrix of the following graph. 6



c) Prove that an arborescence is a tree in which every vertex other than root has an in-degree of exactly one. 6

10. a) Let A_f be the reduced incidence matrix of a connected digraph. Prove that the number of spanning trees in the graph equals the value of det $(A_f . A_f^T)$. 6
b) Prove that determinant of every square submatrix of the incidence matrix of a digraph is 1, –1 or 0. 6
c) Let B and A be respectively the circuit matrix and incidence matrix of a self loop free digraph such that the columns in B and A are arranged using the same order of edges. Prove that $A.B^T = 0$. 6