Reg. No. | | | | | | | | |

BSCMTC 358

Credit Based VI Semester B.Sc. Degree Examination, April/May 2017
(Semester Scheme) (2016-17 Batch Onwards)
MATHEMATICS (Paper – VII)
Partial Differential Equations, Fourier Series and Linear Algebra

Time : 3 Hours Max. Marks : 120

Instructions : *1) Answer **any ten** questions from (Part – **A**). **Each** question carries 3 marks.*

*2) Answer **five full** questions from (Part – **B**) choosing **one full** question from **each** Unit.*

*3) Scientific calculator are **allowed**.*

PART – A **(3×10=30)**

1. Check the integrability of the equation, $(y + z)dx + (z - x)dy = (x + y)dz$.
2. Form a partial differential equation by eliminating a and b form $2z = (ax + y)^2 + b$.
3. Find the complete integral of $(1 - x)p + (2 - y)q = 3 - z$.
4. State the Dirichlet's conditions for Fourier series expansion of a function.
5. Find the half range sine expansion of the function $f(x) = 1$ over the interval $0 < x < 2$.
6. Write the complex form of the Fourier series of a function and formula for complex Fourier coefficients.
7. State whether the space $\{x = (x_1, x_2, \dots x_n) \mid x_1 > x_2, x_i \in R\}$ is a subspace of $\mathbb{R}^n$ over $\mathbb{R}$ or not. Give reason.
8. Determine whether the elements (2, –1, 3) (4, 1, –1) and (2, 3, –3) are linearly independent in $\mathbb{R}^3$ over $\mathbb{R}$.
9. Show that $\|u + v\|^2 = \|u\|^2 + \|v\|^2$ if u and v are real orthogonal vectors.

P.T.O.

10. Let $T : V \to V'$ be a linear transformation. If $v_1, v_2, \ldots v_n$ are elements in V; such that $T(v_1), T(v_2) \ldots T(v_n)$ are linearly independent, show that $v_1, v_2, v_3 \ldots v_n$ are linearly independent.

11. Show that if A is a Nilpotent matrix in $M_n(F)$ then $I + A$ is non-singular.

12. Prove that a linear transformation $T : V \to V$ is a (1, 1) mapping if Ker $T = \{0\}$.

13. Define minimum polynomial of a matrix $A \in M_n(F)$.

14. If $A \in M_n(F)$ is non-singular, then show that the characteristic roots of A^{-1} are inverses of the characteristic roots of A.

15. Show that the system of equations :

$$x_1 - 2x_2 + x_3 = \frac{1}{2}$$
$$2x_1 - 5x_2 + 2x_3 = 1$$
$$x_1 + x_2 + x_3 = 1,$$ has no solution.

PART – B

Unit – I

1. a) Assuming the condition of integrability solve the equation :
$(2xz - yz)dx + (2yz - zx)dy - (x^2 - xy + y^2)dz = 0.$ 6

b) Solve : $(y - z)p + (z - x)q = x - y.$ 6

c) Solve : $p(1 + q^2) = q(z - 1).$ 6

2. a) Assuming the condition of integrability solve the equation :
$(y\cos xy - \sin y)dx + (x\cos xy - x\cos y)dy + 2zdz = 0.$ 6

b) Find the complete integrals of
i) $p^2 + q^2 = 4$ ii) $pq = x.$ 6

c) Solve : $q(p - \sin x) = \cos y.$ 6

Unit – II

3. a) Prove that, $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ if $-\pi \le x \le \pi$ and hence show that
$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$ 9

b) Find the complex form of the Fourier series of the function whose definition in one period is $f(x) = e^{-x}, -1 < x < 1.$ 9

4. a) Find the half range sine and cosine expansion of the function $f(x) = x$ over the interval $0 \leq x < p$. 9

b) Find the Fourier series expansion of the periodic function whose definition in one period is $f(x) = 4 - x^2, -2 \leq x \leq 2$. 9

Unit – III

5. a) i) Define vector space.

ii) Let V be the abelian group of positive real numbers for multiplication. Define scalar multiplication in V by $ax = x^a$, $a \in \mathbb{R}$ and $x \in V$. Show that V is a vector space over the field of real numbers $\mathbb{R}$. 6

b) If the set $\{v_1, v_2, \ldots v_n\}$ is a minimal generating set for a vector space V, then prove that it is a basis of V. 6

c) Let V be an inner product space. Then prove that, $|\langle u, v\rangle| \leq \|u\| \cdot \|v\|$, $u, v \in V$. 6

6. a) i) Define a linearly dependent set in a vector space V.

ii) Show that in a vector space any subset of a linearly independent set is linearly independent. 6

b) Prove that, a vector space V is a direct sum of subspaces V_1 and V_2 if and only if every $v \in V$ can be expressed uniquely as $v = v_1 + v_2$, $v_1 \in V_1$ $v_2 \in V_2$. 6

c) i) Define an inner product space.

ii) Prove that any orthonormal set in an inner product space V is linearly independent. 6

Unit – IV

7. a) i) Define linear transformation of vector spaces. 6

ii) If $T : V \to V'$ is a linear transformation of vector spaces, then prove that Ker(T) is a subspace of V.

b) Let V and V′ be vector spaces of dimension m and n respectively over a field F. Prove that the space L(V; V′) of all linear transformations of V into V′ is isomorphic onto the space $M_{mn}(F)$ of all $m \times n$ matrices over the field F. 6

c) i) Define rank of a linear transformation. 6

ii) Find the rank of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x, 2y, x + y + z)$

8. a) Let V be a finite dimensional vector space over a field F and let W be a subspace of V. Then prove that $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$. 6

b) Show that a linear transformation $T : V \to V'$ of vector space is (1, 1) if and only if it maps any linearly independent set in V onto a linearly independent set in V'. 6

c) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$ be the matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ with respect to the standard basis. Find the matrix m(T) with respect to the basis {(1, 1, 0), (0, 1, 0), (0, 1,1,)} 6

Unit – V

9. a) Find the dimension of the space of solutions of the system :

$2x_1 + x_2 - 3x_3 - x_4 = 0$

$x_1 + 2x_3 = 0$

$3x_1 - x_2 - x_3 + x_4 = 0$. 6

b) Let $A \in M_n(F)$ with $q(x) = a_0 + a_1x + a_2x^2 + \ldots + x^m$ as the minimum polynomial. Prove that A is non-singular if and only if $a_0 \neq 0$. 6

c) Show that $A \in M_n(F)$ is Nilpotent if and only if all its characteristic roots are zero. 6

10. a) Find the inverse of the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 6 \\ 6 & -3 & -1 \end{bmatrix}$ using elementary row operations. 6

b) Let $A \in M_n(F)$ and $q(x) \in F[x]$ be the minimum polynomial of A. If $f(x) \in F[x]$ is any other polynomial satisfied by A, then prove that q(x) divides f(x). 6

c) i) Define characteristic root and characteristic polynomial of a matrix $A \in M_n(F)$.

ii) Find the characteristic roots of the matrix $A = \begin{bmatrix} 2 & 3 & 5 \\ 0 & -1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$ 6

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