Reg. No. | | | | | | | | | |

BSCMTC 355

Credit Based VI Semester B.Sc. Degree Examination, April/May 2017
(2014 – 15 Batch Onwards) (Semester Scheme)
MATHEMATICS – VII
Partial Differential Equations, Fourier Series and Linear Algebra

Time : 3 Hours Max. Marks : 120

***Instructions** : 1) Answer **any ten** questions from (Part – **A**). **Each** question carries **3** marks.*
*2) Answer **five full** questions from (Part – **B**) choosing **one full** question from **each unit**.*
*3) Scientific calculators are **allowed**.*

PART – A

1. Verify the condition of integrability for the equation.
   $(2x^2 + 2xy + 2xz^2 + 1)\,dx + dy + 2zdz = 0.$
2. Solve : $y^2dx - zdy + ydz = 0.$
3. Solve : $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{xy}$.
4. State the Dirichlets condition for a Fourier series expansion of function.
5. Write the Fourier series expansion of even function f(x) and write the formula for a_0, a_n.
6. Find the half range sine expansion of the function $f(x) = 1, 0 < x < 2$.
7. Let V be a vector space over a field F. Prove that :
   i) $a.0 = 0$
   ii) $a.v = 0$ implies either $a = 0$ or $v = 0$.

P.T.O.

8. Determine whether the vectors $v_1 = (1, 0, 1, 0)$, $v_2 = (0, 1, 0, 1)$ and $v_3 = (0, 0, 0, 2)$ of R^4 are linearly independent.

9. Prove that any orthonormal set in V is linearly independent.

10. Prove that the mapping $T : V \to V'$ given by $T(v) = a_1$, where $v = (a_1, a_2, ... a_n) \in F^{(n)}$ is a linear transformation.

11. Let $T : V \to V'$ be the linear transformation defined by $T(x, y, z) = (x', y', z')$ where $x' = 2x$, $y' = 4y$, $z' = 5z$. Find the matrix of T w.r.t. the bases $(2/3, 0, 0)$, $(0, 1/2, 0)$ and $(0, 0, 1/4)$ of R^3.

12. Find the column rank of the matrix.

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 0 & 4 \\ 3 & 2 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

13. Prove that every non-singular matrix is a product of elementary matrices.

14. Prove that the minimum polynomial of A is unique.

15. Define characteristic root of a linear transformation.

PART – B

Unit – I

1. a) Solve : $(e^x y + e^z)\,dx + (e^y z + e^x)\,dy + (e^y - e^x y - e^y z)\,dz = 0$. **6**

   b) Solve by inspection method $(y - z)(y + z - 2x)dx + (z - x)(z + x - 2y)dy + (x - y)(x + y - 2z)\,dz = 0$. **6**

   c) Solve by Homogeneous method $z^2dx + (z^2 - 2yz)dy + (2y^2 - yz - xz)dz = 0$. **6**

2. a) Solve by auxiliary method $zydx = zxdy + y^2dz$. **6**

   b) Solve $\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$. **6**

   c) Solve $(ydx + xdy)(a - z) + xydz = 0$. **6**

Unit – II

3. a) Expand $f(x) = x^2$ in a Fourier series over the interval $(-p, p)$. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$ in $(-\pi, \pi)$. **9**

   b) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$. **9**

4. a) Find the Fourier expansion of the periodic function whose definition in one period is $f(x) = \begin{cases} 0 & \text{if } -\pi \le x \le 0 \\ \sin x & \text{if } 0 \le x \le \pi \end{cases}$. **9**

   b) Find the half range sine expansion of the function $f(x) = x^2$, $0 \le x < 1$. **9**

Unit – III

5. a) Let V be a vector space of dimension n. Prove that any set of m linearly independent elements $(m \le n)$ can be completed to a basis of V. **6**

   b) Prove that the set $\{v_1, v_2, \ldots v_n\}$ is a minimal generating set for V if and only if it is a basis of V. **6**

   c) Prove that V is a direct sum of subspaces V_1 and V_2 if and only if every $v \in V$ can be expressed uniquely as $v = v_1 + v_2$ where $v_1 \in V_1$ and $v_2 \in V_2$. **6**

6. a) Let V be a vector space generated by $v_1, v_2, \ldots v_n$. Prove that $\{v_1, v_2, \ldots v_n\}$ is a basis of V if and only if the set $\{v_1, v_2, \ldots v_n\}$ is a linearly independent set. **6**

   b) Define subspace of a vector space V. Let $V = R^n$ and $W = \{(x_1, x_2, \ldots x_n) \mid x_i \in F\ x_n = 0\}$. Prove that W is a subspace of V. **6**

   c) Define inner product space. Let V be an inner product space. Prove that for any $v, v' \in V$ $|(v\ v')| \le \|v\|\ \|v'\|$. **6**

Unit – IV

7. a) Let V be a vector space over the field F and W be a subspace of V. Let $\{v_1, v_2, \ldots v_n\}$ be a basis of V such that $\{v_1 \ldots v_m\}$ $(m \le n)$ is a basis of W. Prove that $\{\overline{v}_{m+1}, \overline{v}_{m+2} \cdots \overline{v}_n\}$ is a basis of V/W. **6**

   b) Define Kernel of linear transformation. Let $T : V \to V'$ be a linear transformation. Prove that Ker(T) is a subspace of V. **6**

   c) Prove that T is an isomorphism if and only if m(T) is non-singular. **6**

8. a) Let V and V′ be vector space of dimension m and n respectively over a field F. Prove that the space L(V, V′) of all linear transformation of V into V′ is isomorphic onto the space $M_{mn}(F)$ of all m×n matrices over F. 6

b) Let V = V′ = R^3 and let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$ be the matrix of T = L(V, V′) with respect to the standard basis. Find m(T) with respect to the basis (1, 1, 0), (0, 1, 0) and (0, 1, 1). 6

c) Prove that the linear transformation T : V → V′ is a (1, 1) map if and only if Ker T = {0}. 6

Unit – V

9. a) Define minimum polynomial of the matrix A. Let F be a field and let $A \in M_n(F)$. Prove that there exist a non-trivial polynomial $f(x) \in F[x]$ such that f(A) = 0. 6

b) Reduce $A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 6 \\ 6 & -3 & -1 \end{bmatrix}$ to the unit matrix by the elementary column operation and calculate A^{-1}. 6

c) Let $A \in M_n(F)$ with $q(x) = a_0 + a_1x + \ldots + x^m$ as the minimum polynomial of A. Prove that A is non-singular if and only if $a_0 \neq 0$. 6

10. a) Define characteristic root of linear transformation T. Let $A \in M_n(F)$ and let $\lambda \in F$. If λ is a characteristic root of A, then prove that for any $f(x) \in F[x]$, $f(\lambda)$ is a characteristic root of f(A). 6

b) Find the inverse of the matrix A if $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ using elementary row operations. 6

c) Show that the system of equations

$x_1 - 2x_2 + x_3 = \frac{1}{2}$

$2x_1 - 5x_2 + 2x_3 = 1$

$x_1 + x_2 + x_3 = 1$

has no solutions. 6