Reg. No. | | | | | | | | |

BSCMTC 351

Credit Based VI Semester B.Sc. Examination, April/May 2017
(2013 – 14 and Earlier Batches)
MATHEMATICS (Paper – VII)
Partial Differential Equation, Vector Spaces and Real Analysis

Time : 3 Hours Max. Marks : 120

Instructions : *1) Answer **any ten** questions from (Part – **A**). **Each** question carries **3** marks.*
*2) Answer **five full** questions from (Part – **B**), choosing **one full** question from **each** Unit.*
*3) Scientific calculators are **allowed**.*

PART – A **(3×10=30)**

1. Eliminate the arbitrary function from $z = e^y \cdot f(x + y)$.

2. Solve : $pq = k$.

3. Solve the equation $yzp + zxq = xy$.

4. If $v_1, v_2, ..., v_n \in V$ are linearly independent, then prove that every element in their span has a unique representation in the form $\lambda_1 v_1 + \lambda_2 v_2 + ... + \lambda_n v_n$ with $\lambda_i \in F$.

5. If F is the field of real numbers, then prove that the vectors (1, 1, 0, 0), (0, 1, –1, 0) and (0, 0, 0, 3) in $F^{(4)}$ are linearly independent over F.

6. Define an inner product space.

7. Find the matrix which represents the linear transformation described by $(1, 1) \to (0, 1)$ and $(-1, 1) \to (3, 2)$.

8. If the linear transformation $T : V \to W$ is a one-one transformation of V onto W, then prove that its inverse is linear.

P.T.O.

9. Define limit points of a sequence.

10. Show that $\lim_{n\to\infty} \frac{(3n+1)(n-2)}{n(n+3)} = 3$.

11. Show that $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.

12. Show that the series $\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \ldots$ converges for $p > 0$.

13. Prove that the sequence $\frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots + \frac{1}{\lfloor n}$ is convergent.

14. Prove that a necessary condition for the convergence of an infinite series $\sum_{n=1}^{\infty} u_n$ is that $\lim_{n\to\infty} u_n = 0$.

15. State limit comparison test for convergence and divergence of a series of positive terms.

PART – B

Unit – I

1. a) Assuming the condition for integrability, solve $(2x^2 + 2xy + 2xz^2 + 1)\, dx + dy + 2z\, dz = 0$. **6**
   b) Solve : $p^2 + q^2 = x - y$. **6**
   c) Solve : $(y + z)\, p + (z + x)\, q = x + y$. **6**

2. a) Assuming the condition for integrability, solve $(2xz - yz)\, dx + (2yz - zx)\, dy - (x^2 + y^2 - xy)\, dz = 0$. **6**
   b) Solve : $\sqrt{p} + \sqrt{q} = x$. **6**
   c) Solve $p\,(1 + q^2) = q\,(z - a)$. **6**

Unit – II

3. a) Define a linear span of a vector space V and show that it is a subspace of V. **6**

   b) If V is a finite dimensional inner product space over F, then prove that V has an orthonormal set as a basis. **6**

   c) If V is a finite dimensional vector space and W is a subspace of V, then show that dim W $\leq$ dim V. **6**

4. a) If V and W are vector spaces over F. Define Kernel of a homomorphism $T : V \to W$ and prove that it is a subspace of V. **6**

   b) If V is an inner product space over F, then prove that $|\langle u, v\rangle| \leq \|u\| \cdot \|v\|$ for any $u, v \in V$. **6**

   c) If $v_1, v_2, v_3, ..., v_n$ are the basis elements of a vector space V over F and if $w_1, w_2, w_3, ..., w_m$ in V are linearly independent over F, then prove that $m \leq n$. **6**

Unit – III

5. a) Define the product of two linear transformations. Prove that the product of two linear transformations of a vector space V onto itself is linear. **6**

   b) If $T : V \to W$ is any linear transformation, then prove that rank T + nullity T = dim V, where V and W are any two vector spaces. **6**

   c) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix}$ using elementary row operations. **6**

6. a) Prove that an isomorphism T of a vector space V onto a vector space W carries $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_r$ independent vectors of V onto independent vectors in W. **6**

   b) Define a linear transformation. Prove that the mapping $T : V_2(R) \to V_3(R)$ defined by $(x, y)\, T = (x + y, x - y, 2x)$ is a linear transformation. **6**

   c) Using linear transformations find A^{-1} where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. **6**

Unit – IV

7. a) Using the definition of limit of a sequence, prove that $\lim_{n\to\infty} \sqrt[n]{n} = 1$. 6

   b) Prove that the necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded. 6

   c) If $\lim_{n\to\infty} a_n = a$ and $a_n \geq 0$ for all n, then prove that $a \geq 0$. 6

8. a) Prove that a sequence can not converge to more than one limit. 6

   b) Define a Cauchy sequence and show that the sequence $\{S_n\}$ can not converge, where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$. 6

   c) If $\{a_n\}$ and $\{b_n\}$ are two sequences such that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then prove that $\lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n = a \pm b$. 6

Unit – V

9. a) Prove that every absolutely convergent series is convergent. Is the converse true ? Justify your answer. 6

   b) Show that the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ...$ is convergent. 6

   c) Examine the convergence of the series $\frac{1}{\sqrt{1.2}} + \frac{1}{\sqrt{2.3}} + \frac{1}{\sqrt{3.4}} + ...$ 6

10. a) If u_n is a positive term such that $\lim_{n\to\infty} (u_n)^{\frac{1}{n}} = l$, then prove that $\sum_{n=1}^{\infty} u_n$ converges if $l < 1$ and diverges if $l > 1$. 6

    b) Examine the convergence of the series :

    $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + ...$ 6

    c) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1} x^n$. 6
