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# ಮಂಗಳೂರು MANGALORE



# ವಿಶ್ವವಿದ್ಯಾನಿಲಯ UNIVERSITY

ಕುಲಸಚಿವರ ಕಛೇರಿ ಮಂಗಳಗಂಗೋತ್ರಿ - 574199 Office of the Registrar Mangalagangothri - 574199

ದಿನಾಂಕ / Date :

31/5/2014

ಕ್ರಮಾಂಕ / No .: MU/ACC/CR62/2013-14/A2

# NOTIFICATION

Sub: Revised syllabus of Mathematics an optional subject for B.Sc. degree programme.

Ref: Academic Council decision No. 1: 3 (2014-15) dated 24-5-2014.

The revised syllabus of Mathematics an optional subject for B.Sc.

degree programme which was approved by the Academic Council at its meeting held on 24-5-2014 is hereby notified for implementation with effect from the academic year 2014-15.

To:

1) The Principals of the colleges concerned.

2) The Registrar (Evaluation), Mangalore University.

3) The Chairman, UG BOS in Mathematics, Mangalore University.

4) The Superintendent, Academic Section, O/o. the Registrar, Mangalore University.

5) Guard file

#### MANGALORE UNIVERSITY

# Mathematics Syllabus for B. Sc. (Credit Based Semester System) (New Revised Syllabus)

#### PREAMBLE

The Mathematics syllabus for B. Sc. in use at present was introduced from the academic year 2012-2013, by modifying the earlier syllabus, by introducing new text books and reference books. However, due to substantial changes in the syllabus of the pre-university course of Karnataka, introduced from the academic year 2012-2013, the U.G.B.O.S. decided to update the B. Sc. syllabus to keep pace with recent changes in the syllabus of pre-university course. The Board observed that important topics like Group Theory, Number Theory, Complex Analysis etc., are not given proper weightage in the present pre-university syllabus and hence it is necessary to frame a new syllabus for B. Sc. for introduction from the academic year 2014-2015. The following revised syllabus for B.Sc. Mathematics (Credit Based Semester System) of Mangalore University, framed by the U.G.B.O.S., has also taken into consideration the syllabus recommended by the U.G.B.O.S., has also taken into committee and syllabi of other Universities of Karnataka. The syllabus is meant-to-be introduced from the academic year 2014-2015 and it is framed as per the prevailing guidelines of the Credit Based Semester System.

# Course Pattern and Scheme of Examinations

# Group II: Optional III: B.Sc. Mathematics

Semester	Paper	Hours per week	Duration of the Uni. Exam (hrs)	Marks		
				University Exams	Internal Assessment*	Total
I	MT 101 : Paper 1	6	3	120	30	150
11	MT 151 : Paper 2	6	3	120	30	150
111	MT 201 : Paper 3	6	3	120	30	150
IV	MT 251 : Paper 4	6	3	120	30	150
٧	MT 301 : Paper 5	5	3	120	30	150
	MT 302: Paper 6 (Special Paper)**	5	3	120	30	150
VI	MT 351 : Paper 7	5	3	120	30	150
	MT 352 : Paper 8 (Special Paper)**	5	3	120	30	150
*				T	otal	1200

- \* For each paper, the internal assessment marks shall be awarded based on two tests conducted for the purpose.
- \*\* During the Vth & VIth Semesters, a student can opt for any one of the special papers offered in the syllabus.

Semester	Paper	Title of the papers	
I	MT 101 : Paper 1	Number Theory and Calculus	
II	MT 151 : Paper 2	Calculus, Group Theory and Differential Equations	
III	MT 201 : Paper 3	Number Theory, Partial Derivatives and Group Theory	
IV	MT 251 : Paper 4	Multiple Integrals, Complex Variables, Sequences and Series	
	MT 301 : Paper 5	Differential Equations and Ring Theory	
V	MT 302 : Paper 6 (Special paper)	6 a) Discrete Mathematics 6 b) Numerical Analysis	
*	MT 351 : Paper 7	Partial Differential Equations, Fourier Series and Linear Algebra	
VI	MT 352 : Paper 8 (Special paper)	8 a) Graph Theory 8 b) Linear Programming and its Applications	

# QUESTION PAPER PATTERN FOR B.Sc. MATHEMATICS (CREDIT BASED SEMESTER SYSTEM) FOR UNIVERSITY EXAMINATION

- Each Question Paper, for Paper 1 to Paper 8, shall consist of two parts:
   Part A and Part B.
- The number of Questions in each part shall be as tabulated below for different papers:

Papers	Part A Short Answer Questions No. of Questions	Part B Long Answer Questions No. of Questions
Paper 1	15	10
Paper 2	15	10
Paper 3	15	10
Paper 4	15	10
Paper 5	15	10
Paper 6	15	10
Paper 7	15	10
Paper 8	15	10

- Note 1: Fifteen Questions in Part A shall equally cover all the units of the syllabus. Any ten questions shall be answered. Each question in Part A carries three marks for Paper 1 to Paper 8.
- Note 2: In Part B, all papers shall have two questions from each of the five units. Five full questions shall be answered, choosing one full question from each unit. Each question in Part B carries 18 marks for Paper 1 to Paper 8.

# I Semester MT 101:Paper1:Number Theory and Calculus 72 hours: 6 hrs/week

#### Unit 1 (15 hrs)

Number Theory: Division algorithm, the greatest common divisor, Euclidean algorithm, Diophantine equation, the fundamental theorem of arithmetic.

Text Book: Elementary Number theory by David M Burton, 6<sup>th</sup> Edition-Tata McGraw Hill

Chapter 2: Sections 2.2, 2.3, 2.4, 2.5

Chapter 3: Sections 3.1, 3.2

#### Unit 2 (14hrs)

Concavity- Curve Sketching: Definition of Concavity, Point of inflections, Second derivative test for local Extrema, graphing, applied optimization problems

Text Book: Thomas' calculus, by Maurice D. Weir, Joel Hass and Frank R. Giordano, 11<sup>th</sup> Edition, Pearson Publications, 2008.

Chapter 4: 4.4, 4.5

# Unit 3 (14 hrs)

Limits of Finite Sums: Riemann sum, definite integral, Limits of Riemann sums, Integrable and non-integrable functions. Area under the graph of a non-negative function. Average value of continuous function. Mean value theorem for definite integrals. Fundamental theorem for definite integral.

Text Book: Thomas' calculus, by Maurice D. Weir, Joel Hass and Frank R. Giordano,

11th Edition, Pearson Publications, 2008.

Chapter 5: Section 5.2, 5.3, 5.4

### Unit 4 (14 hrs)

**Techniques of Integration:** Products of powers of sines and cosines, tanx and cotx. Trigonometric substitution. Reduction formulas, numerical integration, trapezoidal rule.

Text Book: Thomas' calculus, by Maurice D. Weir, Joel Hass and Frank R. Giordano, 11th Edition, Pearson Publications, 2008.

Chapter 8: Section 8.4, 8.5, 8.7

#### Unit 5(15 hrs)

Conic sections and quadratic equations: Definitions, parabolas, Ellipses, Hyperbolas, classifyconic section by eccentricity.

Quadratic equations and rotations, possible graphs of Quadratic equations, discriminant tests.

Text Book: Thomas' calculus, by Maurice D. Weir, Joel Hass and Frank R. Giordano, 11th Edition, Pearson Publications, 2008.

Chapter 10: Section 10.1, 10.2, 10.3

Reference books: (1) Number Theory by H.S. Hall, S.R. Knight, Maxford Books, 2008. (2) Calculus with Analytical Geometry by Louis Leithold, 5<sup>th</sup> edition, Harper and Row Publishers, New York, 1986.

#### II Semester

# MT151:Paper2:Calculus, Group Theory and Differential Equations 72 hours; 6 hrs/week

# Unit 1(15 hrs)

Mean value theorem: Rolle's Theorem, mean value theorem. Indeterminate forms and L'Hospital's rule: Indeterminate form  $\frac{0}{0}$ , L'Hospital's rule (first form), L'Hospital's rule(stronger form), Cauchy's mean value theorem, indeterminate forms  $\frac{\infty}{\infty}$ ,  $\infty \cdot 0$ ,  $\infty - 0$ ; Taylor's theorem, estimating the remainder. Polar co-ordinates: definition, relating polar and Cartesian co-ordinates, graphing in polar co-ordinates; symmetry, tests for symmetry, slope of curves, tracing curves. Areas and Lengths in polar co-ordinates, Area in the plane, area between curves, length of a polar curve.

Text Book: Thomas' calculus, by Maurice D.Weir, Joel Hass and Frank R.Giordano, 11th edition, Pearson publications, 2008.

Chapter 4: Section 4.2, 4.6

Chapter 10: Section 10.5, 10.6, 10.7

# UNIT 2 (15hrs)

# Applications of definite integrals:

Volumes by slicing and Rotation about axis: Definition of volume, calculating the volume of a solid, volume of a pyramid, volume of a wedge, solids of revolution:the disk method, washer method.

Volume by cylindrical shell method: finding the volumes by using shells, the shell method;rotation about y-axis and x-axis

Length of plane curves: Length of a parametrically defined curve-definition and derivation of formula for the length of y=f(x)

Text Book: Thomas' calculus, by Maurice D.Weir, Joel Hass and Frank R.Giordano, 11th edition, Pearson publications, 2008.

Chapter 6: Section 6.1, 6.2, 6.3

# UNIT 3 (14 hrs)

Group Theory:

Introduction, binary operation, groups, subgroups, cyclic groups, permutation

Text Book: University algebra by N. S. Gopalakrishnan - revised second edition, New Age

International - 2009

Chapter 1: Section 1.1, 1.2, 1.3, 1.4, 1.5, 1.11

### UNIT 4 (14 hrs)

# Differential equation:

Variable separable and homogeneous equations. Exact equations, linear equation of order one,integrating factors found by inspection, determination of integrating factors, Bernoulli's equation, co-efficients linear in the two variables.

Text Book: A Short Course in Differential equations by Earl D.Rainville and Phillip E.Bedient,4th edition,IBM publishing company,Bombay 5.(1969)

Chapter 3: Section 10,11

Chapter 4: Section 18,19,21,22

#### UNIT 5 (14 hrs)

### Applications of differential equations

Velocity of escape from the earth, Newton's law of cooling, simple chemical conversions, orthogonal trajectories-rectangular co-ordinates, orthogonal trajectories-polar co-ordinates.

### Non-linear equations:

Factorizing the left member, singular solutions, the c-discriminant equations, thep-discriminant equation, eliminating the dependent variable, Clairaut's equation, dependent variable missing, independent variable missing.

Text Book: A Short Course in Differential equations by Earl D.Rainville and Phillip E.Bedient,4<sup>th</sup> edition,IBM publishing company,Bombay 5(1969).

Chapter 3: Section 13,14,15,16,17

Chapter 16: Section 82,83,84,85,86,87,88,89

Reference books:

- (1) Calculus with Analytical geometry by Louis Leithold,5<sup>th</sup>edition, Harper and Row publishers, Newyork, 1986.
- (2) Topics in algebra by I.N.Herstein,2<sup>nd</sup> edition,John Wily & sons,2007.
- (3)Differential Equations with Applications and programs by S.BalachandraRao and H.R.Anuradha, University Press, 2009.

#### III Semester

# MT201:Paper3:Number Theory, Partial Derivatives and Group Theory 72 hours; 6 hrs/week

# Unit 1 (14 hrs)

The Theory of Congruences, Properties of Congruences, Binary and Decimal representation of integers, Linear Congruences and the Chinese Remainder theorem.

Text Book : Elementary Number Theory by David M. Burton - VI Edition

Chapter 4 : Sections 4.2, 4.3, 4.4.

#### UNIT 2 (14 hrs)

Fermat's Theorem, Wilson's Theorem, Euler's Phi-Function, Euler's Theorem, Some properties of Phi-Function, Finite continued fractions.

Text book : Elementary Number Theory by David M. Burton - VI Edition

Chapter 5 : Sections 5.2, 5.3

Chapter 7 : Sections 7.2, 7.3, 7.4

Chapter 15: Section 15.2

#### UNIT 3 (15 hrs)

#### Partial Derivatives

Functions of several variables: Definition of function of n independent variables, Domains and ranges, , Functions of 2 variables, Definition of interior and boundary points, Definitions of open , closed , bounded and unbounded regions in a plane.

Graphs, level curves, and contours of functions of 2 variables, Level curves, graph, surface, Functions of three variables, Level surface, Interior and boundary points for space regions, open and closed regions.

Limits and continuity in higher dimensions: Limits and continuity. Two path test for non-existence of limit, continuity of composites, Functions of more than two variables, Extreme values of continuous functions on closed and bounded sets.

Partial derivatives: Partial Derivative of a function of two variables, implicit partial differentiation, finding slope of a surface in the y-direction, Functions of more than two variables, Partial derivatives and continuity, Second Order partial derivatives, Mixed Derivative theorem, Partial Derivatives of still higher order, Differentiability: Increment theorem for functions of two variables, Differentiable function, corollary.

Chain Rule: Chain rule for functions of two and three independent variables-Functions defined on surfaces, Implicit differentiation, Exercise 14.4 on page 1003:1,

Text Book: Thomas' calculus, by Maurice D.Weir, Joel Hass and Frank R.Giordano, 11th edition, Pearson publications, 2008.

Chapter 5 : 14.1, 14.2, 14.3, 14.4

#### UNIT 4 (15 hrs)

Directional derivatives and Gradient vectors: Directional derivatives in the plane- Definition, Interpretation of the directional derivative, Gradient vector: Properties of the directional derivatives, Gradients and tangents to level curves, Rules for gradients, Gradients of functions of three variables.

Tangent planes and Differentials: Tangent planes and normal lines, Equation of a plane tangent to a surface, Linearising a function of two variables, Definition of standard linear approximations, Differentials: Total differentials, Linearisation and total differentials of functions of more than two variables.

Extreme values and saddle points: Derivative tests for Local Extreme values:Local maxima and minima, First Derivative test for local extreme values, critical and saddle points, Absolute Maxima and Minima and closed bounded regions,

#### Constrained Maxima and Minima

Text Book: Thomas' calculus, by Maurice D.Weir, Joel Hass and Frank R.Giordano, 11<sup>th</sup> edition, Pearson publications, 2008.

Chapter 14: Section 14.5, 14.6, 14.7, 14.8

# UNIT 5 (14 hrs)

Lagrange's theorem, Euler's theorem, Fermat's theorem, Isomorphism, Klein 4- group, automorphism, Homomorphism, kernel of homomorphism, Normal subgroups, Subgroups of index 2.

Text Book: University Algebra By N.S. GopalakrishnanNew Age International Publishers (2009) .

Chapter 1 :Section 1.6, 1.7, 1.8

#### Reference books:

- (1) Number Theory by H.S. Hall, S.R. Knight Maxford Books, 2008.
- (2) Calculus with Analytical Geometry by Louis Leithold, 5th edition, Harper and Row publisher, New York, 1986.
- (3) Topics in algebra by I.N. Herstien, 2nd edition, John Wily & Sons. 2007.

#### IV Semester

# MT251:Paper4:Multiple Integrals, Complex Variables, Sequences and Series

72 hours; 6 hrs/week

Unit 1(15 hrs)

### Multiple Integrals

Double integrals: Doubles Integrals over rectangles, Double Integrals as volume, The Fubini's Theorem (First Form), Double Integrals over bounded non-rectangular Regions, Fubini's Theorem (Stronger Form), Finding Limits of integration, Properties of double integrals. Reversing the order of integration, Volume beneath a surface.

Areas of bounded regions in plane : Definition of area, examples

Double integrals in Polar form : Integrals in Polar coordinates, Finding limits of Integration, Changing Cartesian Integrals into Polar Integrals.

Triple Integrals in Rectangular Coordinates: Volume of a region in space, Definition, Finding limits of integration, Properties of Triple Integrals.

Text Book: Thomas' calculus, by Maurice D. Weir, Joel Hass and Frank R. Giordano,

11th Edition, Pearson Publications, 2008.

Chapter 15: Section 15.1, 15.2, 15.3, 15.4

### UNIT 2 (14 hrs)

# Complex variables

Polar and Exponential Forms, Powers and roots, Functions of a Complex variable, Limits, Continuity, Differentiability, Cauchy Riemann Equations, Analytic functions, Entire functions.

Text Book: Complex variables theory and applications II Edition by H.S. Kasana, PHI Learning Private Limited, New Delhi(2008).

Chapter 1: Section 1.3, 1.4

Chapter 2: Section 2.1, 2.2, 2.3, 2.4, 2.5, 2.6

#### UNIT 3 (14 hrs)

Harmonic functions, Elementary functions: Exponential function, Trigonometric functions, Hyperbolic functions and Logarithmic functions. Complex integration – Contour integral.

Text Book: Complex variables theory and applications II Edition by H.S. Kasana, PHI Learning Private Limited, New Delhi. (2008).

Chapter 2: Section 2.7

Chapter 3: Section 2.1, 2.2, 2.3, 2.4, 2.5, 2.6

Chapter 4: Section 4.1, 4.2

#### UNIT 4 (15 hrs)

Infinite Sequences and Series: Infinite Sequences: Definitionsof infinite sequence, Convergence and Divergence, Limit, Definition of Divergence to Infinity, Calculating limits of sequences, Sandwich theorem for sequences, The Continuous Function Theorem for Sequences, Convergence of a sequence using L'Hospital's Rule, Theorem Bounded non decreasing sequences- Definitions of bounded non decreasing sequences, bounded sequences, Upper bound, Least upper bound.

Infinite Series: Definition of Infinite series, nth term, partial sum, Convergence and sum of the series, Geometric series, nth term test for divergence, Combining Series,

Taylor's and Maclaurin's Series: Series representations, Definitions, Taylor and Maclaurin's series, Taylor Polynomials- Definition of Taylor polynomial of order n.

Taylor's Theorem: Taylor's formula.

Text Book: Thomas' calculus, by Maurice D. Weir, Joel Hass and Frank R. Giordano,

11th Edition, Pearson Publications, 2008.

Chapter 11: Section 11.1, 11.2, 11.8, 11.9

# UNIT 5 (14 hrs)

Convergence/ Divergence tests for infinite series.

The integral Test: Non decreasing Partial sums: The Integral Test.

Comparison test: Limit Comparison test, The Ratio and Root Tests

Alternating series - absolute and conditional convergence: The Alternating SeriesTest:Leibnitz'sTheorem, Absolute and conditional convergence, The Absolute Convergence Test- The Rearrangement Theorem for Absolutely Convergent series.

Text Book: Thomas' calculus, by Maurice D. Weir, Joel Hass and Frank R. Giordano,

11th Edition, Pearson Publications, 2008.

Chapter 11: Section 11. 3, 11.4, 11.5, 11.6

#### Reference books:

- (1) Calculus with Analytical Geometry by Louis Leithold, 5th edition, Harper and Row publisher, New York, 1986.
- (2) Complex Variables and Applications by James Ward Brown and Ruel V. Churchill, 7th Edition, McGraw Hill Publications, 2003.

# V Semester – Paper 5 MT301:Paper5:Differential Equations and Ring Theory 60 hours; 5 hrs/week

# Unit 1 (12 hrs)

Linear equation with constant coefficients: Definition, operator D, complementary function of a linear equation with constant coefficients. Particular integral, General method of finding particular integral, Special methods for finding particular integral when RHS of the non-homogenous Differential equation is of the form: (i)  $e^{ax}$  (ii) sinax (iii) cosax (iv)  $x^m$ 

Text Book: Differential Equations: S.Narayanan ManicavachagomPillay. Viswanathan (Printers and Publishers)PVT LTD 1985 Revised Ninth Edition.

#### UNIT 2 (12 hrs)

Special methods for finding particular integral when RHS of the non-homogeneous Differential equation is of the form  $e^{ax}$  where (i) v = sinax (ii) v = cosax (iii)  $v = x^m$ . Linear equations with variable coefficients.

Special methods to solve any second order equation:

i) Reduction to normal form ii) Change of independent variable. iii) Reduction of order iv) Variation of parameters

Text Book: i) Differential Equations:S Narayanan& ManicavachagomPillay.

Viswanathan (Printers and Publishers)PVT LTD 1985 Revised
Ninth Edition.

ii) A short course in Differential equations: Earl D Rainville and Philip E Bedient(1969)

#### UNIT 3 (12hrs)

The Laplace transform: Definition, transforms of elementary functions, transforms of derivatives. Derivatives of transforms the gamma function periodic functions.

Inverse transforms: Definition, a step function, Convolution theorem, simple initial value problems. Application to spring problem, Vibration of a spring damped and undamped vibrations.

Text Book: A short course in differential equations: Earl D Rainville and Philip E. Bedient(1969).

# UNIT 4 (12 hrs)

Ring Theory: Definition of Rings, Unit Element, Commutative Ring.

Integral domains: Zero divisors, Integral domain, Field, Division ring (Skew field), regular elements, Finite Integral domains, Center of a ring.

Ring Homorphisms: Homomorphism and Kernel of a ring homomorphism.

Isomorphism: Isomorphism, Embedding

Ideals: Definition of ideals, Simple Rings, Left and right ideals, Sum and Product of two ideals.

Quotient rings: Definition, First Isomorphism Theorem.

Text Book: University Algebra by N. S. Gopalakrishnan- revised 2<sup>nd</sup>Edition NewAge International(2009).

Chapter 2: Section 2.2., 2.3, 2.4, 2.5, 2.7, 2.8

# UNIT 5 (12 hrs)

Prime and Maximal Ideals Prime Ideals, Prime ideals in Z, Maximal Ideals Factorization: Divisibility, Associates, Irreducible elements, Prime elements, g.c.d., Relatively prime elements.

Euclidian Domain: Definition, Examples, Existence of g.c.d., Factorization Theorem.

Polynomial Rings: Polynomials, Polynomial rings, Degreeof apolynomial, Constant polynomial, Irreducible polynomials.

Text Book: University Algebra by N. S. Gopalakrishnan- revised 2<sup>nd</sup>edition NewAge International(2009).

Chapter 2: Section 2.9, 2.10, 2.11, 2.14

#### Reference books

- (1) Topics in Algebra by I. N. Herstein, 2nd Edition, John Wily & Sons, 2007.
- (2) Differential Equations with Applications and programs by S. BalachandraRao and H. R. Anuradha, Universities Press(2009).

# V Semester MT302,Paper6a;Special Paper – DISCRETE MATHEMATICS 60 hours; 5 hrs/week

# UNIT 1 (12 hrs)

Partially Ordered sets & Lattice Theory: Definition and examples of partially ordered sets. Lattices: Set theoretic & Algebraic definitions, Examples for lattices, Duality principle, Sub-lattices & Convex sub-lattices, Ideals of lattices, Complements & Relative complements, Homomorphism & Isomorphism, Distributive and Modular lattices, Characterization of distributive and modular lattices in terms of sub-lattices.

#### UNIT 2 (12 hrs)

Graphs and Planar Graphs: Introduction, Basic terminology, Multigraphs and Weighted graphs, Digraphs and relations, Representation of graphs, Operations on graphs, Paths and circuits, Graph traversals, Eulerian paths and circuits, Hamiltonian paths and circuits, Factor of a graph, Planar graphs, Graph colouring.

#### UNIT 3 (12 hrs)

Trees and Cut-sets: Trees, Rooted trees, Path lengths in rooted trees, Prefix codes, Spanning trees and cut-sets, Minimum spanning trees; Kruskal's Algorithm, Prim's Algorithm.

# UNIT 4 (12 hrs)

Modeling Computation: Introduction, Russell's Paradox and Noncomputability, ordered sets, Languages, Phrase structure grammars, Types of grammars and languages, Basic concepts of Information processing machine, finite state machines, Finite state machines as models of physical systems, Equivalent machines, Finite state machines as language recognizers.

Analysis of Algorithms: Introduction, Algorithms LARGEST1, LARGEST2, BUBBLESORT and LARGESMALL algorithms, Time complexity of algorithms, Tractable and Intractable problems.

# UNIT 5 (12 hours)

Discrete numeric functions and Generating functions: Introduction, Manipulation of numeric functions, Asymptotic behaviour of numeric functions, Generating functions.

Recurrence relations and Recursive Algorithms: Introduction, Recurrence relations, Linear recurrence relation with constant coefficients, Homogeneous solutions, particular solutions.

#### Text Books:

[1] Elements of Discrete Mathematics 3<sup>rd</sup>edition by C.L. Liu, Tata Macgraw Hill Publishers(2008).

[2]Introduction to Lattice Theory by Gabor Szasz, Academic Press, New York and London, 1963.

#### Reference books

- Discrete Mathematical Structures with Applications to Computer Science by J.P. Trembley and R. Manohar, TataMagrawHill Publishers.
- (2) Discrete Mathematics for Computer Scientists by J. K. Truss, Pearson Education Asia.

# V Semester MT302,Paper6b:Special Paper – NUMERICAL ANALYSIS 60 hours; 5 hrs/week

UNIT 1 (12 hrs)

# Applications and Errors in Computation:

Introduction, accuracy of numbers, errors, useful rules for estimating errors, error propagations, error in the approximation of a function. Errors in a series approximation.

# Solutions of Algebraic and Transcendental Equations:

Introduction: Initial approximation, rate of convergence, Bisection method, method of false position or Regulafalsi method, Iteration method, Newton Raphson method.

Chapter 1: Section 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7.

Chapter 2: Section 2.1, 2.5(1), 2.6, 2.7, 2.8, 2.10.2.11

UNIT 2 (12 hrs)

### Solution of Simultaneous Algebraic Equations:

Introduction to matrices- Definition, Special matrices, Operation on matrices, Related matrices, Rank of a matrix, Elementary transformations of a matrix, Equivalent matrix, Consistency of a system of linear equation, System of linear homogeneous equations.

Solution of linear homogeneous equations, Direct methods of solution – matrix inversion method, Gauss eliminationmethod, Gauss-Jordan method.

Iterative methods of solution- Jacobi's iterationmethod, Gauss-Seidel iteration method.

Chapter 3: Sections 3.2,3.3,3.4, 3.5.

UNIT 3 (12 hrs)

#### Finite differences:

Introduction, Finite differences, differences of a polynomial, to find one or more missing terms.

Interpolation:Introduction, Newton's forward interpolation formula, Newton's backward interpolation formula, Interpolation with unequal intervals, Lagrange's interpolation formula.

Chapter 6: Sections 6.1,6.2,6.3, 6.8.

Chapter 7: Sections 7.1,7.2,7.3,7.11, 7.12.

# UNIT 4 (12 hrs)

Divided differences: Newton's divided difference formula, Inverse interpolation, Lagrange's method.

Numerical differentiation and integration: Numerical differentiation, Formulae for derivatives- Derivatives using forward difference formulae, Derivatives using backward difference formulae. Maxima andminima of a tabulated function.

Numerical integration: Newton cotes quadrature formulae, Trapezoidal rule, Simpson's one-third rule, Simpson's three-eighth rule.

Chapter 7: Sections 7.13,7.14,7.19, 7.20

Chapter 8: Sections 8.1,8.2,8.3,8.4,8.5

UNIT 5 (12 hrs)

# Numerical Solution of Ordinary Differential Equations:

Introduction, Picard's method, Taylor's series method, Euler's method, Modified Euler's method, Runge-kutta method, Predictor-corrector methods, Adams-Bashforth method.

Chapter 10: Sections 10.1,10.2,10.3,10.4,10.5,10.7,10.8, 10.10.

Text Book: Numerical methods in Engineering and Science with programs in C, C++ by Dr. B .S. GREWAL, Ninth edition, April 2010, KhannaPublications, New Delhi.

#### Reference books:

 Introductory Methods of Numerical Analysis by S. S. Sastri, 3<sup>rd</sup> Edition, Prentice Hall of India(2008).

# VI Semester MT351:Paper7:Partial Differential Equations, Fourier Series and Linear Algebra

60 hours; 5 hrs/week

# UNIT 1 (12 hrs)

Total Differential equations and Partial differential equations, Criterian of integrability, Rule for integrating Pdx + Qdy + Rdz = 0, Solution of Pdx + Qdy + Rdz = 0.

Formation of partial differential equations by eliminating constants and by eliminating arbitrary functions. Lagranges method of solving linear equations Pp + Qq = R.

Non-linear equations of the type:

 $f_2(v,q)$ 

i) 
$$F(p,q) = 0$$
 ii)  $a)F(x,p,q) = 0$ b)  $F(y,p,q) = 0$ iii)  $f_1(x,p) =$ iv)  $x = px + qy + f(p,q)$ 

Text Book: Differential Equations: Narayanan & ManicavachagomPillay, S. Viswanathan (Printers and Publishers) PVT LTD 1985 Revised Ninth Edition.

#### UNIT 2 (12 hrs) Fourier Series

Introduction, Periodic functions, Euler's Formulae, Definite integrals. Dirichlet's conditions for a Fourierseries expansion, Even and Odd functions, Half Range Series, Complex Fourier Coefficients, Finite Fourier Transforms.

Text Book: Differential Equations with Applications and Programs by S. BalachandraRao and H. R. Anuradha, Universities Press(2009).

Chapter 15: Section 15.2, 15.3, 15.4, 15.5, 15.6, 15.8.

Unit 3 (12 hrs)

# Linear Algebra

**Vector Spaces:** Properties, Subspaces Intersection of subspaces, L(S)-Subspace generated by a subset, Nature of elements of L(S), Sum of subspaces, Direct sum of two subspaces, Characterization of direct sum, Direct sum of n subspaces.

Linear Dependence, Independence and Bases: Basis, Generating set, Linear independence, Minimal generating set, Dimension, Dimensions of subspaces, Dimension of a sum of subspaces.

Inner Product Spaces: Inner product, Norm, Schwarz inequality, Orthogonal vectors, Normal vectors, Orthonormal basis and linear independence of orthonormal sets, Existence of orthonormal basis in an inner product space, Orthogonal complements.

Text Book: University Algebra by Gopalakrisnan- 2<sup>nd</sup> revised edition, New Age International(2009)

Chapter 3: Section 3.2, 3.3, 3.4

Chapter 5: Section 5.11

# Unit 4 (12 hrs)

Linear Transformations: Linear transformation, Kernel, Isomorphism, Isomorphism of  $F^{(n)}$  with any n-dimensional space, Quotient space, First Isomorphism Theorem, dimension of a quotient space, non-singular transformation, L(V,V'), dimension of L(V,V').

Matrices: Identity, Idempotent, Nilpotent, Non-singular, Diagonal, Triangular and Block Matrices.

Matrices and Linear transformations: Matrix associated with a linear transformation, Isomorphism of L(V,V') with  $M_{mm}(F)$ , Matrix of a product of linear transformations, Relation between matrices of a Linear Transformation with respect to different bases, Similar matrices.

Rank: Row rank, Column rank, Rank of a matrix, Rank of a linear transformation, Rank of a composition of linear transformations, Rank of a product of matrices.

Text Book: University Algebra by Gopalakrisnan- 2<sup>nd</sup> revised edition, New Age International(2009)

Chapter 3: Section 3.5.

Chapter 5: Section 5.2, 5.3, 5.5

### Unit 5 (12 hrs)

Elementary Row Operations: Elementary matrices, Non-singularity of elementary matrices, Inverse of an elementary matrix, Inverse of a matrix as a product of elementary matrices, Equivalent matrices.

Linear Equations: Homogeneous linear Equations, Condition for existence of non-trivial solutions, Non-homogeneous Equations, condition for existence of solutions and five conditions for the existence of a unique solution.

Minimal polynomial:Definition and existence of Minimal polynomial, Uniqueness, Minimal polynomial of non-singular matrices, minimum polynomial of similar matrices, Minimal polynomial of a transformation.

Characteristic roots: Characteristic roots of f(A) for a polynomial f and matrix A, number of distinct Characteristic Roots, Characteristic polynomial of a matrix, Characteristic polynomial of similar matrices, Characteristic polynomial of a linear transformation, Cayley- Hamilton theorem, Characteristic polynomial of the transpose.

Text Book: University Algebra by Gopalakrisnan- 2<sup>nd</sup> revised edition, New Age International(2009)

Chapter 5: Section 5.5, 5.6, 5.8, 5.9

#### Reference books:

(1) Topics in Algebra by I. N. Herstein.

(2) A short course in Differential Equations by Earl D. Rainvelleand Philip E. Bedient.

# VI Semester MT352: Paper 8(a) Special Paper – Graph Theory 60 hours; 5 hrs/week

# Unit 1 (12 hrs)

Graph ,finite, Infinite graphs, Incidence and degree , Isolated vertex, Pendent vertex, Null graph, Isomorphism, Sub-graphs, Walks, Paths, Circuits, Connected and disconnectedgraphs, Components, Euler graphs, Operation on graphs, Hamiltonian paths, Circuits, Trees and some properties of trees, Rooted and binary tree, Spanning tree and fundamental circuits.

Section: 1.1, 1.2, 1.3, 1.4, 1.5, 2.1, 2.2, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.1, 3.2, 3.3, 3.4, 3.5, 3.7, 3.8

# Unit 2 (12 hrs)

Cutsets, Properties, Fundamental cut sets, Connectivity, Seperability, Planar graphs, Kuratowski's graphs, Different representation of planar graphs, Geometric dual

Section: 4.1, 4.2, 4.3, 4.4, 4.5, 5.2, 5.3, 5.4, 5.6

## Unit 3 (12 hrs)

Ring sum of two circuits, Subspace, Orthogonal vectors, Matrix representation, Incidence matrix, Cutset matrix, Circuit matrix, Adjacency matrix.

Section: 6.1, 6.4, 6.5, 6.7, 6.8, 7.1, 7.2, 7.3, 7.4, 7.6, 7.9

# Unit 4 (12hrs)

Chromatic number and Chromatic polynomial.

Section: 8.1, 8.3

### Unit 5 (12 hrs)

Directed graph, Types, Matrices in graphs, Enumeration of graphs, Counting labelled trees. Section: 9.1, 9.2, 9.4, 9.8, 10.1, 10.2

Text Book: Graph theory With Applications to Engineering and Computer Science by Narsingh Deo, PHI Learning Private Limited.

#### VI Semester

MT352: Paper 8(b) Special Paper – Linear Programming and its Applications 60 hours; 5 hrs/week

# UNIT 1 (12 hrs)

Geometric Linear Programming: Profit Maximization and cost Minimization, Cost Minimization, Canonical forms for Linear Programming Problems, Polyhedral Convex sets.

The Simplex Algorithm: Canonical stack forms for Linear Programming Problems, Tucker Tableaus, Pivot Transformation, Pivot Transformation for Maximum and Minimum Tableaus, Simplex Algorithm for Maximum Basic Feasible Tableaus., Simplex Algorithm for Maximum Tableau.

Chapter 1: Section 1, 2, 3. Chapter 2: Section 1, 3, 5.

# UNIT 2 (12 hrs)

Negative Transposition: The Simplex Algorithm for Minimum tableaus.

Non-Canonical Linear Programming problems: Unconstrained variables, Equations of Constraint.

Duality Theory: Duality in Canonical Tableaus, Dual Simplex Algorithm, Matrix formulation of Canonical Tableaus, The Duality Equation.

Chapter 2: Section 7 Chapter 3: Section 1, 2

Chapter 4: Section 1, 2, 3, 4.

#### UNIT 3 (12 hrs)

The Duality Theorem: Duality in Non-Canonical Tableaus.

Matrix Games: Two Persons Zero Sum Matrix Game, Linear Programming Formulation of Matrix Games, The Von Neumann Minimax Theorem.

Chapter 4: Section 5, 6. Chapter 5: Section 1, 2, 3.

#### UNIT 4 (12 hrs)

Transportation and Assignment Problem: The Balanced Transportation Problem, The Vogel Advanced Start Method (VAM), The Transportation Algorithm, Unbalanced Transportation Problems, The Assignment Problem, The Hungarian Algorithm.

Chapter 6: Section 1, 2, 3, 5, 6.

# UNIT 5 (12hrs)

Network- Flow Problems: Graph Theoretic Preliminaries, The Maximal Flow Network Problems, The Max-Flow Min-Cut Theorem, The Maximal Flow Algorithm, The Shortest Path Network Problem- The Shortest Path Algorithm I.

Chapter 7: Section 1, 2, 3, 4

Text Book: Linear Programming and its Applications by James K Strayer, Narosa Publishing House, Springer International.

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# Model Question Paper

# Credit Based First Semester B.Sc. Degree Examination,

# MATHEMATICS

# Number Theory and Calculus

Time: 3 Hours

Max. Marks: 120

Instructions:

Answer any ten questions from Part A.

- Answers to Part A should be written in the first few pages of the main
- Answer five full questions from Part B choosing one full question

#### PART-A

(3x10 = 30)

- State division algorithm for numbers.
- 2. If  $a \mid bc$  and gcd(a,b) = 1, then prove that  $a \mid c$ .
- 3. Sate whether the following Diophantine equations can be solved or not
  - i) 6x + 51y = 22
- ii) 33x+14y=115
- 4. Determine the point of inflection and concavity of the function  $f(x) = x^{1/3}$ .
- 5. Find absolute maximum and absolute minimum function  $f(x) = x^3 + 5x - 4$  in [-3,1]. the
- 6. Find the vertical asymptotes of the graph of the function  $f(x) = \frac{4x^2}{x^2-5}$
- 7. Find the average value of the function  $f(x) = -3x^2 1$  on [0,1].
- 8. Use Leibnitz rule to find the derivative of f(x) if  $f(x) = \int_{-L}^{1} \frac{1}{dt} dt$
- 9. Find where the function  $f(x) = 3 \frac{3x}{2}$  on [0,2] takes the Average Value in its domain.
- 10. Obtain the reduction formula for  $\int (\ln x)^n dx$
- Evaluate ∫ sin(lnx) dx
- 12. Using trapezoidal rule with n = 4, find  $\int x^2 dx$ .
- 13. State the equations that result in rotation of the coordinate axis in a Cartesian
- 14. Find the angle of rotation for the conic  $2x^2 + \sqrt{3}xy + y^2 10 = 0$  in order to remove the xy term.
- 15. State the discriminant test for determining the type of the conic by an equation  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$

#### PART B

#### UNIT-I

- 1. a) Given integers a and b no both of which are zeros prove that there exist x and y such that gcd(a,b) = ax + by.
  - b) If a = qb + r, prove that gcd(a,b) = gcd(b,r).
  - c) Prove that linear Diophantine equation ax + by = c has solution if and only if  $d \mid c$  where  $d = \gcd(a, b)$  and if  $x_0$  and  $y_0$  are solutions, then prove that  $x = x_0 + \frac{b}{d}t$  and  $y = y_0 \frac{b}{d}t$ . (6+6+6)
- 2. a) Solve the Diophantine equation 172x + 20y = 100.
  - b) Prove that every positive integer x>1 can be expressed as a product of primes.
  - c) A customer bought a dozen pieces of fruit- apples and oranges for \$1.32. If an apple costs 3 cents more than an orange, and more apples were purchased than oranges how many pieces of each kind were bought? (6+6+6)

#### UNIT - II

- 3. a) State and prove second derivative test for local maximum.
  - b) A card board box manufacturer wishes to make open boxes from pieces of card board 12 in square by cutting equal squares from four corners and turning up the sides. Find the length of the sides of the square to be cut out to obtain a box of largest possible volume.
  - c) If  $f(x)=(1-2x)^3$  find the point of inflection of the graph of f(x) and determine where the graph is concave upward and concave downward. (6+6+6)
- 4. a) A rectangular field is to be fenced off along the bank of a river; no fence is required along the river. If the material for the fence costs \$8 per running foot for the two ends and \$12 per running foot for the side parallel to the river, find the dimension of the field of largest possible area that can be enclosed with \$3600 worth of fence.
  - b) Find all the asymptotes of the graph of the function  $f(x) = \frac{x^2 \delta}{x 3}$ .
  - c) Sketch the graph of the function  $f(x)=x^3-3x^2+3$ . (6+6+6)

#### UNIT - III

- 5. a) If f is continuous at every point of [a,b] and F is an antiderivative of f on [a,b] then prove that  $\int_a^b f(x) = F(b) F(a)$ .
  - b) Find the area of the region under the line y=x over the interval [0,b],b>0 using Riemann Sum.
  - c) Find the upper sum obtained by dividing the interval into n equal subintervals and calculate the area under the curve  $f(x) = x^2 + 1$  on [0,3]. (6+6+6)
- 6. a) If f is continuous on [a,b] then at some point c in [a,b], prove that  $f(c) = \frac{1}{b-c} \int_{c}^{b} f(x) dx$ 
  - b) If f is continuous on [a,b] prove that  $F(x) = \int_a^x f(t)dt$  is continuous on [a,b] and differentiable on (a,b) and show that F'(x) = f(x).
  - c) Find the derivatives of

i) 
$$\int_{1}^{x^{2}} cost \ dt$$
 ii)  $\int_{0}^{x} \sqrt{1-t^{2}} \ dt$  (6+6+6)

#### UNIT -IV

- 7. a) Obtain the reduction formula for  $\int \cos^n x \, dx$  and hence find  $\int \cos^3 x \, dx$ .
  - b) Evaluate  $\int \sin^4 x \cos^4 x \, dx$
  - c) Obtain the reduction formula for  $\int tan^n x \, dx$  and hence evaluate  $\int tan^5 x \, dx$  (6+6+6)
- 8. a) Derive the reduction formula for  $\int s(n^n x \cos^m x dx)$ 
  - b) Evaluate

i) 
$$\int \frac{x^2 + x + 1}{(x^2 + 1)^2} dx$$
 ii)  $\int e^{\sqrt{3x + 9}} dx$ 

c) Evaluate  $\int \frac{\sqrt{x}}{1+\sqrt{x}} dx$  (6+6+6)

#### UNIT- V

- 9. a) Find the foci, vertices, centre and eccentricity of the ellipse  $\frac{(y-4)^2}{16} + \frac{(y-1)^2}{3} = 1$ 
  - b) Determine the expression for discriminant test for determining the type of a general second degree equation in x and y.
  - e) Find new equation of the conic 2xy=9 by rotating the axis through an angle of n/4 about the origin.
- 10. a) Rotate the coordinate axis to change the equation  $3x^2 + 2\sqrt{3}xy + y^2 8x + 8\sqrt{3}y = 0$  in to an equation that has no xy term.
  - b) Reduce the equation  $2x^2 + \sqrt{3}xy + y^2 10 = 0$  to a standard form without xy and linear terms.
  - c) Find centre, foci, vertices, asymptotes of the conic  $2x^2-y^2+6y=3$ . (6+6+6)

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# **Model Question Paper**

Credit Based Second Semester B.Sc. Degree Examination,

# MATHEMATICS

# Calculus, Group Theory and Differential Equations

Time: 3 Hours

Max. Marks: 120

Instructions:

1. Answer any ten questions from Part A.

- Answers to Part A should be written in the first few pages of the main answer book.
- Answer five full questions from Part B choosing one full question from each Unit.

PART - A

(3x10 = 30)

- 1. Find a suitable value of c satisfying the conclusion of the Mean value theorem for the function  $f(x) = x + \frac{1}{x}$  on  $\left[\frac{1}{2}, 2\right]$
- 2. Evaluate  $\lim_{x \to \frac{\pi}{2}} \frac{\sec x}{1 + \tan x}$
- 3. Find the Cartesian equation of the polar curve  $r = \frac{5}{\sin \theta 2\cos \theta}$ .
- 4. Find the volume of the solid generated by revolving the region between the curve.
- 5. Using the shell method, find the volume of the solid generated by revolving the region bound by the curves  $x = \sqrt{y}$ , x = -y and the line y = 2.
- 6. Find the length of the curve, x = 1 t, y = 2 + 3t,  $-\frac{2}{3} \le t \le 1$ .
- 7. If G is a group such that  $a^2 = e$  for every  $a \in G$ , then show that G is abelian.
- 8. Give an example to show that the union of two subgroups may not be a subgroup.
- 9. Express the inverse of the cycle (1 2 4 5 3) as a product of transpositions.
- 10. Check for the exactness of the equation  $(2xy + y)dx + (x^2 + x) dy = 0$ .
- 11. Solve :  $y^1 = x 2y$ .
- 12. Find the integrating factor of the equation y(x + y)dx + (x + 2y 1)dy = 0
- 13. Find the orthogonal trajectories of the family of curves  $x^2 y^2 = c$ .
- 14. solve  $x^2p^2 y^2 = 0$ ,
- 15. Solve :  $y = px + p^3$

#### PART B

#### UNIT-1

- 1. a) State and prove Rolle's theorem.
  - b) By using L'Hospital's rule evaluate :
    - (i)  $\lim_{x\to 0} \left( \frac{1}{\sin x} \frac{1}{x} \right)$
    - (ii)  $\lim_{x\to\infty} \left(x-\sqrt{x^2+x}\right)$
  - c) Find the area of the region that lies inside the circle r = 1 and outside the cardioid  $r = 1 \cos \theta$ . (6+6+6)
- a) Suppose functions f and g are continuous on [a, b] and also suppose g'(x) ≠ 0
  throughout (a, b). Then prove that there exists a number C ∈ (a, b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

- b) Graph the curve  $r^2 = 4 \cos \theta$ .
- c) Find the length of the cardiod =  $r(a + 1 \cos \theta)$ .

#### (6+6+6)

#### UNIT - II

- 3. a) Find the volume of the solid generated by revolving the region between the parabola  $x = y^2 + 1$  and the line x = 3 about the line x = 3.
  - b) The region bounded the curve  $y = \sqrt{4x x^2}$ , the x-axis, and the line x = 2 is revolved about the x-axis to generate a solid. Find the volume of the solid by using shell method.
  - c) If c: x = f(t), y = g(t),  $a \le t \le b$ , where  $f^l$  and  $g^l$  are continuous but not simultaneously zero on [a, b] and C is traversed exactly once as t increases from a to b, then derive the formula for the length of C in the form

$$L = \int_{a}^{b} \sqrt{[f^{\dagger}(t)]^{2} + [f^{\dagger}(t)^{2}]} dt$$
 (6+6+6)

- 4. a) Find the length of the asteroid  $x = \cos^3 t$ ,  $y = \sin^3 t$ ,  $0 \le t \le 2\pi$ .
  - b) The region bounded by the curve  $y = \sqrt{x}$ , the x-axis and the line x = 4 is revolved about the y-axis to generate a solid. Find the volume of the solid.
  - c) The region bounded by the parabola  $y = x^2$  and the line y = 2x in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid. (6+6+6)

#### UNIT - III

- a) Prove that a non-empty subset H of a group G in a subgroup if and if whenever a ∈ H, b ∈ H, the product ab<sup>-1</sup> ∈ H.
  - b) Let H and K be subgroups of a group G. Then prove that HK is a subgroup of G if and only if HK = KH.
  - c) Prove that every σ ∈ S<sub>n</sub>, where S<sub>n</sub> is the set of all permutations on n symbols, can be expressed as a product of disjoint cycles. (6+6+6)
- a) Let H be a finite subset of a group G such that a.b ∈ H, ∀ a, b ∈ H. Then prove that H is a subgroup of G.
  - b) Let G be a finite group of order n. Then prove that G is isomorphic to a subgroup of S<sub>n</sub>, where S<sub>n</sub> is the set of all permutations on n symbols.
  - c) Let H and K be finite subgroups of G such that HK is also a subgroup. Then prove that  $O(HK) = \frac{O(H) \cdot O(K)}{O(H \cap K)}$ . (6+6+6)

#### UNIT-IV

- 7. a) Solve:  $(1 + y^2 + xy^2) dx + (x^2y + y + 2xy)dy = 0$ .
  - b) Solve:  $y(y^3 x)dx + x(y^3 + x)dy = 0$
  - c) Solve: (x + 2y 4) dx (2x + y 5) dy = 0. (6+6+6)
- 8. a) Solve:  $(y \cos^2 x) dx + \cos x dy = 0$ 
  - b) Solve:  $(4xy + 3y^2 x)dx + x(x + 2y)dy = 0$
  - c) Solve:  $6y^2 dx x(2x^3 + y) dy = 0$  (6+6+6)

#### UNIT- V

- 9. a) A thermometer reading 18°F is brought into a room, the temperature of which is 70°F. One minute later the thermometer reading is 31°F. Determine the temperature reading as a function of time and in particular, find the temperature reading 5 minutes after the thermometer is first brought into the form.
  - b) Find the orthogonal trajectories of the family of cardioides  $r = a(1 + \cos \theta)$ .
  - c) Solve the differential equation:

$$xp^2 - 3yp + 9x^2 = 0$$
, for  $x > 0$ . (6+6+6)

- 10. a) Solve the differential equations  $xyp^2 + (x + y)p + 1 = 0$ .
  - b) A bacterial population B is known to have a rate of growth proportional to B itself. If between noon and 2 p.m, the population triples, at what time, no controls being exerted, should B become 100 times what it was at noon?

c) Solve: 
$$xy^{11} - (y^1)^3 - y^1 = 0$$
. (6+6+6)

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